Mixed $\mathscr{K}$-Dissipativity and Stabilization to ISS for Impulsive Hybrid Systems

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LiLab

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- Dissipativity

Introduction

⏭ Dissipativity

J. C. Willems, "Dissipative dynamical systems, Part 1: General theory" *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 321–351, 1972.

J. C. Willems, "Dissipative dynamical systems, Part 2: Quadratic supply rates" *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 359–393, 1972.

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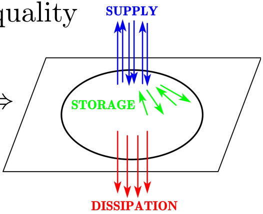
J. C. Willems, "Dissipative dynamical systems, Part 2: Quadratic supply rates" *Arch. Rational Mech. Anal.*, vol. 45, no. 5, pp. 359–393, 1972.

DI: Dissipativity inequality

DI: { supply rate; storage function } ⇒



耗散： supply − storage

Introduction

- Hybrid systems
  discrete time、continuous time

Introduction

- Hybrid systems
  discrete time、continuous time

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(a, q)$$

- x_1: position
- x_2: speed
- a: acceleration input
- $q \in \{1, 2, 3, 4, 5, -1, 0\}$: gear

Introduction

⏭ stability analysis

- Method: dissipativity of subsystems
- Issue: dissipativity under multiple supply rates

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> A. R. Teel, "Asymptotic stability for hybrid systems via decomposition, dissipativity, and detectability," in *Proc. 49th IEEE Conf. Decis. Control*, Atlanta, GA, USA, 2010, pp. 15–17.

mixed dissipativity ⇒ the stability of hybrid systems.

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- Limitation: systems without external disturbances

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mixed dissipativity ⇒ the stability of hybrid systems.

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E. D. Sontag, "Smooth stabilization implies coprime factorization," *IEEE Trans. Autom. Control*, vol. 34, no. 4, pp. 435–443, Apr. 1989.

input-to-state stability (ISS)⇒ how external inputs or disturbances affect a system's stability

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⏭ Mixed $\mathscr{K}$-Dissipativity and Stabilization to ISS for Impulsive hybrid systems (IHSs)

- dissipativity
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①*mixed supply rate* ②*mixed $\mathscr{K}$-dissipativity*

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⇓

the stabilization to ISS for an IHS

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①***mixed supply rate*** ②***mixed $\mathscr{K}$-dissipativity***

⇓

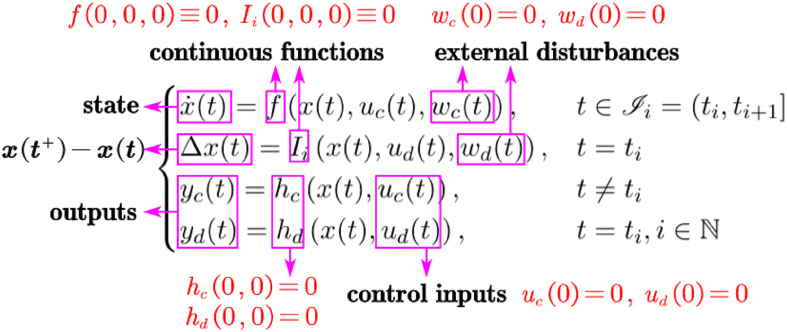
the stabilization to ISS for an IHS

⇓

case of impulsive interconnected network

Preliminaries and $\mathscr{K}$-Dissipativity

Consider a IHS with external disturbances as



$$
\begin{cases}
\dot{x}(t) = f(x(t), u_c(t), w_c(t)), & t \in \mathscr{I}_i = (t_i, t_{i+1}] \\
\Delta x(t) = I_i(x(t), u_d(t), w_d(t)), & t = t_i \\
y_c(t) = h_c(x(t), u_c(t)), & t \neq t_i \\
y_d(t) = h_d(x(t), u_d(t)), & t = t_i, i \in \mathbb{N}
\end{cases} \tag{1}
$$

$x(t_i^+) = \lim_{t \to t_i^+} x(t)$、 $\{t_i\}_{i\in\mathbb{N}}$: impulsive sequence、$\Delta_i = t_{i+1} - t_i$: impulsive interval

Mixed $\mathscr{K}$-Dissipativity for IHS

Mixed $\mathscr{K}$-Dissipativity for IHS

Mixed supply rate

A function (γ_c, γ_d) is called a ***mixed supply rate*** of IHS (1) if γ_c is locally integrable and γ_d is locally summable, i.e.

$$\left| \int_{t_0}^{t} \gamma_c \left(u_c(s), y_c(s) \right) ds \right| < +\infty$$

$$\left| \sum_{k=0}^{i} \gamma_d(u_d(t_k), y_d(t_k)) \right| < +\infty$$

Mixed $\mathscr{K}$-Dissipativity for IHS

Mixed $\mathscr{K}$-dissipative

IHS is said to be ***mixed $\mathscr{K}$-dissipative*** under (γ_c, γ_d) if there exist storage functions V_c, V_d and functions $r_c, r_d \in \mathscr{K}_\infty$ such that,

$$D^+V_c(x) \leq \gamma_c(u_c, y_c) + r_c(\|w_c(t)\|), \quad t \in \mathscr{I}_i, i \in \mathbb{N} \tag{1}$$

$$\Delta V_d(x) \leq \gamma_d(u_d, y_d) + r_d(\|w_d(t)\|), \quad t = t_i, i \in \mathbb{N} \tag{2}$$

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1) D^+: Dini derivative; 2) $\Delta V_d(x(t))$: $V_d(x(t^+)) - V_d(x(t))$.

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A function $\gamma : \mathbb{R}_+ \to \mathbb{R}_+$

1. class-$\mathscr{K}$: it is continuous, zero at zero, and strictly increasing.

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$V_c = V_d$

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Special case

$V_c = V_d \Rightarrow$ ***$\mathscr{K}$-dissipative***

Specialization to $\mathscr{K}$-Dissipativity

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Consider a general input–output system without impulse.

$$\begin{cases} \dot{x} = f(x(t), u(t), w(t)) \\ y = h(x, u), t \geq t_0 \end{cases} \tag{3}$$

Specialization to $\mathscr{K}$-Dissipativity

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$$\begin{cases} \dot{x} = f(x(t), u(t), w(t)) \\ y = h(x, u), t \geq t_0 \end{cases} \tag{3}$$

$\mathscr{K}$-dissipativity

System (3) is said to have ***$\mathscr{K}$-dissipativity*** w.r.t. supply rate γ if there exists a storage function V and $\phi \in \mathscr{K}$ such that

$$D^+V(x) \leq \gamma(u, y) + \phi(\|w\|), \quad t \geq t_0 \tag{4}$$

Specialization to $\mathscr{K}$-Dissipativity

$$D^+V(x) \leq \gamma(u, y) + \phi(\|w\|), \quad t \geq t_0$$

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Specialization to $\mathscr{K}$-Dissipativity

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$$V(x_{k+1}) - V(x_k) \leq S(y_k, u_k)$$

Special case

$w = 0 \Rightarrow$ classical dissipativity $V(x_{k+1}) - V(x_k) \leq \gamma(u,y)$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

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Input-to-state stability (ISS)

IHS (1) is said to be ***ISS*** if there exists functions $\beta \in \mathscr{K}\mathscr{L}$ and $\tilde{r}_c, \tilde{r}_d \in \mathscr{K}_\infty$ such that, for any $k \in \mathbb{N}$, we have

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \tilde{r}_c(\|w_c\|_{[t]}) + \tilde{r}_d(\|w_d\|_{[t]}), \quad t \geq t_0.$$

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1. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$
   - class-$\mathscr{K}\mathscr{L}$: $\beta(\cdot, t)$ is of class-$\mathscr{K}$ for $t \geq 0$ and $\beta(s, \cdot)$ is monotonically decreasing to zero for $s > 0$.

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$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \tilde{r}_c(\|w_c\|_{[t]}) + \tilde{r}_d(\|w_d\|_{[t]}), \quad t \geq t_0.$$

1. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$
   - class-$\mathscr{K}\mathscr{L}$: $\beta(\cdot, t)$ is of class-$\mathscr{K}$ for $t \geq 0$ and $\beta(s, \cdot)$ is monotonically decreasing to zero for $s > 0$.
2. A function $w : \mathbb{N} \to \mathbb{R}^n$
   - $\|w\|_{[t]} = \sup_{t_0 \leq s \leq t}\{\|w(s)\|\}$ for all $t \geq t_0 \geq 0$.

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

Lemma 1

$\forall\ a, b \in \mathbb{R}_+\ \forall\ \gamma \in \mathscr{K},\ \forall\ \rho \in \mathscr{K}_\infty$ with $\rho - \mathbf{1} \in \mathscr{K}_\infty$, we have

$$\gamma(a+b) \leq \gamma(\rho(a)) + \gamma(\rho \circ (\rho - \mathbf{1})^{-1}(b))$$

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$$\gamma(a+b) \leq \gamma(\rho(a)) + \gamma(\rho \circ (\rho - \mathbf{1})^{-1}(b))$$

Assumption 1

$$1 < \Delta_{\inf} \triangleq \inf_{i\in\mathbb{N}}\{\Delta_i\} \leq \Delta_{\sup} \triangleq \sup_{i\in\mathbb{N}} < \infty \tag{5}$$

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Assumption 1

$$1 < \Delta_{\inf} \triangleq \inf_{i\in\mathbb{N}}\{\Delta_i\} \leq \Delta_{\sup} \triangleq \sup_{i\in\mathbb{N}} < \infty \tag{5}$$

Assumption 2

$\exists\ \psi_1, \psi_2 \in \mathscr{K}_\infty$ and constants $\lambda_i, \mu_i, i \in \mathbb{N}$ $(0 < \lambda_i \leq \lambda,\ 0 < \mu_i \leq \mu)$, for some $\lambda > 0$, $\mu > 0$, such that

$$\psi_1(\|x\|) \leq V(x) \leq \psi_2(\|x\|), \quad V = V_c, V_d \tag{6}$$

$$V_c(t_i^+) \leq \lambda_i V_d(t_i^+),\ V_d(t_i) \leq \mu_i V_c(t_i), \quad i \in \mathbb{N}. \tag{7}$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

Theorem 1

Assume IHS is ***mixed $\mathscr{K}$-dissipative*** w.r.t. (γ_c, γ_d) and V_c, V_d satisfying (1), (2), (6) and (7), and assume the following

1. $\exists\ \varphi_c\ (\varphi(0) = 0)$, $p_i\ (|p_i| \leq p)$, for some $p > 0$ such that
$$\gamma_c\left(\varphi_c(x(t)), h_c(x(t))\right) \leq p_i V_c(t), \quad t \in \mathscr{I}_i \tag{8}$$
2. $\exists\ \varphi_d\ (\varphi_d(0) = 0)$, q_i, with $q_i > -1$ such that
$$\gamma_d\left(\varphi_d(x(t)), h_d(x(t))\right) \leq q_i V_d(t), \quad t = t_i,\ i \geq 1 \tag{9}$$
3. $\exists\ \alpha > 0$ such that
$$(\alpha + p_i)\Delta_i + \ln(1 + q_i) + \ln(\lambda_i \mu_i) \leq 0, \quad i \in \mathbb{N} \tag{10}$$

$\Rightarrow$ IHS is stabilized to ISS under the state feedback control law as
$$(u_c, u_d) = (\varphi_c(x), \varphi_d(x)). \tag{11}$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

mixed $\mathscr{K}$-dissipativity: $D^+V_c(x) \leq \gamma_c + r_c(\|w_c(t)\|), \Delta V_d(x) \leq \gamma_d + r_d(\|w_d(t)\|)$
condition 1: $\gamma_c \leq p_i V_c(t)$ **condition 2**: $\gamma_d \leq q_i V_d(t)$

$$\Rightarrow D^+V_c(x) \leq p_i V_c(t) + r_c(\|w_c(t)\|), \quad t \in \mathscr{I}_i$$
$$V_d(t^+) \leq (1+q_i)V_d(t) + r_d(\|w_d(t)\|), \quad t = t_i$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

mixed $\mathscr{K}$-dissipativity: $D^+V_c(x) \le \gamma_c + r_c(\|w_c(t)\|), \Delta V_d(x) \le \gamma_d + r_d(\|w_d(t)\|)$
condition 1: $\gamma_c \le p_i V_c(t)$ **condition 2**: $\gamma_d \le q_i V_d(t)$

$$\Rightarrow D^+V_c(x) \le p_i V_c(t) + r_c(\|w_c(t)\|), \quad t \in \mathscr{I}_i$$
$$V_d(t^+) \le (1+q_i)V_d(t) + r_d(\|w_d(t)\|), \quad t = t_i$$

$$|p_i| \le p \Rightarrow R_1 \triangleq \begin{cases} \frac{1}{p_i} e^{p_i(t-t_i)-1} & p_i > 0 \\ t - t_i & p_i \le 0 \end{cases} \le \max\left\{\frac{1}{p} e^{p\Delta_{\text{sup}}} - 1, \Delta_{\text{sup}}\right\}$$

$$\Rightarrow V_c(t) \le V_c(t_i^+) e^{p_i(t-t_i)} + R_1(p_i, t-t_i) r_c(\|w_c\|_{[t]}) \tag{12}$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

$V_d(t^+) \le (1+q_i)V_d(t) + r_d(\|w_d(t)\|)$
Assumption 2: $V_c(t_i^+) \le \lambda_i V_d(t_i^+),\ V_d(t_i) \le \mu_i V_c(t_i)$

$$\Rightarrow V_c(t_{i+1}) \le \lambda_i \mu_i (1+q_i) e^{p_i(t-t_i)} V_c(t_i) + R_1(p_i, t-t_i) r_c(\|w_c\|_{[t]}) + \lambda_i e^{p_i(t-t_i)} r_d(\|w_d(t_i)\|), \quad t \in \mathscr{I}_i \tag{13}$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

$V_d(t^+) \le (1+q_i)V_d(t) + r_d(\|w_d(t)\|)$
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$$\Rightarrow V_c(t_{i+1}) \le \lambda_i\mu_i(1+q_i)e^{p_i(t-t_i)}V_c(t_i) + R_1(p_i, t-t_i)r_c(\|w_c\|_{[t]}) + \lambda_i e^{p_i(t-t_i)}r_d(\|w_d(t_i)\|), \quad t \in \mathscr{I}_i \tag{13}$$

$a_i \triangleq V_c(t_i),\ \ \sigma_i \triangleq p_i\Delta_i + \ln(1+q_i) + \ln(\lambda_i\mu_i),\ \ X_i \triangleq e^{\sigma_i},\ \ Y_i \triangleq R_1(p_i, \Delta_i),\ \ Z_i \triangleq \lambda_i e^{p_i\Delta_i}$

$$\Rightarrow a_{i+1} \le X_i a_i + Y_i r_c(\|w_c\|_{[t_{i+1}]}) + Z_i r_d(\|w_d\|_{[t_i]}) \tag{14}$$

□

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

$$\underset{\sim}{a_{i+1} \leq X_i a_i + Y_i r_c(\|w_c\|_{[t_{i+1}]}) + Z_i r_d(\|w_d\|_{[t_i]})}$$

$$\begin{aligned} a_{i+1} &\leq X_i\{X_{i-1}a_{i-1} + Y_{i-1}r_c(\|w_c\|_{[t_i]}) + Z_{i-1}r_d(\|w_d\|_{[t_{i-1}]})\} \\ &\quad + Y_i r_c(\|w_c\|_{[t_{i+1}]}) + Z_i r_d(\|w_d\|_{[t_i]}) \leq \cdots \\ &\leq e^{\sum_{j=0}^{i}\sigma_j} V_c(t_0) + \sum_{j=0}^{i} G_i(X, Y_j) r_c(\|w_c\|_{[t_{i+1}]}) \\ &\quad + \sum_{j=0}^{i} G_i(X, Z_j) r_d(\|w_d\|_{[t_i]}) \end{aligned} \tag{15}$$

where $G_i(X, H_i) = H_i$, $G_i(X, H_j) = e^{\sum_{k=j+1}^{i}\sigma_k} H_j$, $0 \leq j \leq i-1$, $H = Y, Z$.

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

$\because$ **Assumption 1** : $0 < \Delta_{\text{inf}} \leq \Delta_{\text{sup}} < +\infty$, **condition 3**: $\alpha > 0$
$\therefore \exists M > 0$ such that $\sum_{j=0}^{i} e^{-\alpha(t_{i+1}-t_{j+1})} < M$

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证明.

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$\therefore \exists M > 0$ such that $\sum_{j=0}^{i} e^{-\alpha(t_{i+1}-t_{j+1})} < M$

$G_i(X, H_j) = e^{\sum_{k=j+1}^{i} \sigma_k} H_j,$

$$\Rightarrow \sum_{j=0}^{i} G_i(X, Y_j) \leq K_1, \quad \sum_{j=0}^{i} G_i(X, Z_j) \leq K_2 \tag{16}$$

where $K_1 = \max\left\{\frac{1}{p}e^{p\Delta_{\sup}} - 1, \Delta_{\sup}\right\} M$, and $K_2 = \lambda e^{p\Delta_{\sup}} M$. □

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

证明.

$$\Rightarrow a_i \leq e^{-\alpha(t_i - t_0)} V_c(t_0) + K_1 r_c(\|w_c\|_{[t_i]}) + K_2 r_d(\|w_d\|_{[t_i]}) \quad (17)$$

Stabilization to ISS for Mixed-$\mathscr{K}$-Dissipativity IHS

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For any $t \in \mathscr{I}_i$, by (17), (12), (13), (6), and using Lemma 1, it is easy to see that $\exists \tilde{r}_c, \tilde{r}_d \in \mathscr{K}_\infty$

$$\|x(t)\| \leq \beta(\|x_0\|, t - t_0) + \tilde{r}_c(\|w_c\|_{[t]}) + \tilde{r}_d(\|w_d\|_{[t]})$$

where $\beta(r, s) = \psi_1^{-1}\left(\lambda\mu q e^{(\alpha+p)\Delta_{\sup}} e^{-\alpha s}\psi_2(r)\right) \in \mathscr{K}\mathscr{L}$.
Hence, IHS (1) is stabilized to ISS under the state feedback control law (11). □

Case of Impulsive Interconnected Networks

Case of Impulsive Interconnected Networks

Consider the following impulsive interconnected network:

$$S_i : \begin{cases} \dot{x}_i = A_i x_i + B_i w_i, & t \in \mathscr{I}_j \\ \Delta x_i = C_i x_i + D_i(u_i + F_i w_i), & t = t_j \\ y_i = E_i x_i, \quad t \geq 0; \quad i = 1, 2; & j \in \mathbb{N} \end{cases} \tag{18}$$

subject to the following interconnection control:

$$u_1 = K_1 y_2, \quad u_2 = K_2 y_1, \quad t \geq 0 \tag{19}$$

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$$u_1 = K_1 y_2, \quad u_2 = K_2 y_1, \quad t \geq 0 \tag{19}$$

Denote $\bar{u}_i = u_i + F_i w_i$, the quadratic supply rates as

$$\phi_i(\bar{u}_i, y_i) = \bar{u}_i^\top R_i \bar{u}_i + 2\bar{u}_i^\top S_i y_i + y_i^\top Q_i y_i \tag{20}$$

Case of Impulsive Interconnected Networks

Theorem 2

$\exists P_i > 0$, $a_i \geq 0$ and $b_i > 0$, $i = 1, 2$, such that the following LMIs hold:

$$\begin{pmatrix} P_iA_i + A_i^\top P_i - a_iP_i & P_iB_i \\ B_i^\top P_i & -b_iI \end{pmatrix} \leq 0 \tag{21}$$

$$\begin{pmatrix} \tilde{C}_i^\top P_i\tilde{C}_i - E_i^\top Q_iE_i - P_i & \tilde{C}_i^\top P_iD_i - E_i^\top S_i^\top \\ D_i^\top P_i\tilde{C}_i - S_iE_i & D_i^\top P_iD_i - R_i \end{pmatrix} \leq 0 \tag{22}$$

where $\tilde{C}_i = I + C_i$. Then, IHSs (18) are ***mixed $\mathscr{K}$-dissipative*** w.r.t. $(\gamma_{ci}, \gamma_{di})$, where $\gamma_{ci} = a_ix_i^\top P_ix_i$, and $\gamma_{di} = \phi_i(\bar{u}_i, y_i)$.

Case of Impulsive Interconnected Networks

证明.

Case of Impulsive Interconnected Networks

证明.

Let $V_i(x_i) = x_i^\top P_i x_i$, by (21) and (22), we get

$$D^+V_i(t) \leq a_i V_i(t) + b_i \|w_i(t)\|^2, \ t \neq t_j \tag{23}$$

$$\Delta V_i(t) \leq \phi_i(\bar{u}_i, y_i), \ t = t_j, j \in \mathbb{N} \tag{24}$$

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$$D^+V_c(x) \leq \gamma_c(u_c, y_c) + r_c(\|w_c(t)\|), \quad t \in \mathscr{I}_i, i \in \mathbb{N}$$
$$\Delta V_d(x) \leq \gamma_d(u_d, y_d) + r_d(\|w_d(t)\|), \quad t = t_i, i \in \mathbb{N}$$

$\Rightarrow$ The ***mixed $\mathscr{K}$-dissipative*** of every node in (18) can be directly proofed.

Case of Impulsive Interconnected Networks

证明.

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⇒ The ***mixed $\mathscr{K}$-dissipative*** of every node in (18) can be directly proofed.

Note: $a_i \geq 0$, it is allowed that both systems $\dot{x}_i = A_i x_i + B_i w_i$ have no ISS.

Case of Impulsive Interconnected Networks

Theorem 3

$\exists c_i, d_i, c > 0$ satisfying $0 < c_1 < 1$, $0 < c_2 < c$, such that,

$$\begin{pmatrix} E^\top \Phi_1 E + \bar{c}\bar{P} & E^\top \Phi_2 F \\ F^\top \Phi_2^\top E & F^\top \Phi_3 F - \bar{d} I \end{pmatrix} \leq 0 \tag{25}$$

$$\max\{a_1, a_2\} \cdot \Delta_{\sup} + \ln\left(1 - \max\left\{c_1, \frac{c_2}{c}\right\}\right) < 0 \tag{26}$$

$\Phi_1 = \begin{pmatrix} Q_1 + cK_2^\top R_2 K_2 & S_1^\top K_1 + cK_2^\top S_2 \\ K_1^\top S_1 + cS_2^\top K_2 & cQ_2 + K_1^\top R_1 K_1 \end{pmatrix}$, $\Phi_3 = \mathrm{diag}\{R_1, cR_2\}$,
$\Phi_2 = \begin{pmatrix} S_1^\top & cK_2^\top R_2 \\ K_1^\top R_1 & cS_2^\top \end{pmatrix}$, $\bar{P} = \mathrm{diag}\{P_1, P_2\}$, $\bar{c} = \mathrm{diag}\{c_1 I, c_2 I\}$,
$\bar{d} = \mathrm{diag}\{d_1 I, d_2 I\}$, $E = \mathrm{diag}\{E_1, E_2\}$, and $F = \mathrm{diag}\{F_1, F_2\}$.
Then the network (18) is stabilized to the ISS under interconnection control (19).

Case of Impulsive Interconnected Networks

证明.

Case of Impulsive Interconnected Networks

证明.

Denote $x = (x_1^\top, x_2^\top)^\top$, let $V(x) = V_1(x_1) + cV_2(x_2)$, then we have

$$\psi_1(\|x\|) \le V(x) \le \psi_2(\|x\|) \tag{27}$$

$\psi_1(s) = \min\{\lambda_{\min}(P_1), c\lambda_{\min}(P_2)\}s^2, \psi_2(s) = \max\{\lambda_{\max}(P_1), c\lambda_{\max}(P_2)\}s^2$

Case of Impulsive Interconnected Networks

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$\psi_1(s) = \min\{\lambda_{\min}(P_1), c\lambda_{\min}(P_2)\}s^2, \psi_2(s) = \max\{\lambda_{\max}(P_1), c\lambda_{\max}(P_2)\}s^2$Let $p = \max\{a_1, a_2\}$, $q = -\max\{c_1, \frac{c_2}{c}\}$, by (21), (22), (25), (26), we get

$$\Rightarrow D^+V(t) \le pV(t) + b_1 \|w_1\|^2 + cb_2 \|w_2\|^2, \quad t \ne t_j \tag{28}$$

$$\begin{aligned} \Delta V(t) &\le \phi_1(\bar{u}_1, y_1) + c\phi_2(\bar{u}_2, y_2) \\ &\le qV(t) + d_1 \|w_1\|^2 + d_2 \|w_2\|^2, \quad t = t_j, j \in \mathbb{N} \end{aligned} \tag{29}$$

Case of Impulsive Interconnected Networks

证明.

Denote $x = (x_1^\top, x_2^\top)^\top$, let $V(x) = V_1(x_1) + cV_2(x_2)$, then we have

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$$\Rightarrow D^+V(t) \leq pV(t) + b_1 \|w_1\|^2 + cb_2 \|w_2\|^2, \quad t \neq t_j \tag{28}$$

$$\begin{aligned} \Delta V(t) &\leq \phi_1(\bar{u}_1, y_1) + c\phi_2(\bar{u}_2, y_2) \\ &\leq qV(t) + d_1 \|w_1\|^2 + d_2 \|w_2\|^2, \quad t = t_j, j \in \mathbb{N} \end{aligned} \tag{29}$$

By (28), (29), and Theorem 1, $\Rightarrow$ all the conditions of Theorem 1 are satisfied $\Rightarrow$ network (18) is stabilized to the ISS under (19). □

Example

Consider a network (18), where

$$A_1 = \begin{pmatrix} 0.01 & 0 & 0.1 \\ 0 & 0.1 & 0.2 \\ 0 & 0 & 0.1 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} -0.2 & 0 & 0 \\ 0 & 0.1 & 0.1 \\ 0.1 & 0 & 0.05 \end{pmatrix}$$

$$B_1 = B_2 = I, \qquad C_1 = C_2 = -0.95I$$

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Let the matrices of quadratic supply rates (20) be

$$Q_1 = \begin{pmatrix} -0.0315 & 0.0004 & 0.0251 \\ 0.0004 & -0.0001 & 0.0002 \\ 0.0251 & 0.0002 & -0.0315 \end{pmatrix} \quad S_1 = -0.3Q_1, \quad R_1 = -4Q_1$$

$$Q_2 = \begin{pmatrix} -0.1239 & -0.0001 & 0.0029 \\ -0.0001 & -0.0005 & 0.0002 \\ 0.0029 & 0.0002 & -0.0219 \end{pmatrix} \quad S_2 = -0.3Q_2, \quad R_2 = -4Q_2.$$

Example

By solving LMIs (21) and (22) in Theorem 2, we get $a_1 = 0.22 > 0$, $a_2 = 0.21 > 0$, $b_1 = 1.3$, $b_2 = 1.0$, and

$$P_1 = \begin{pmatrix} 0.1050 & -0.0014 & -0.0836 \\ -0.0014 & 0.0.0002 & -0.0007 \\ -0.0836 & -0.0007 & 0.1050 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0.4131 & 0.0002 & -0.0097 \\ 0.0002 & 0.0017 & -0.0006 \\ -0.0097 & -0.0006 & 0.0730 \end{pmatrix}.$$

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By Theorem 2, every node is ***mixed $\mathscr{K}$-dissipative*** w.r.t. $(\gamma_{ci}, \gamma_{di})$, where $\gamma_{ci} = a_i x_i^\top P_i x_i$, and $\gamma_{di} = \phi(\bar{u}_i, y_i)$.

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Note: $\dot{x}_i(t) = A_i x_i(t) + B_i w_i(t)$, $i = 1, 2$, have no ISS.

Example

$$u_1 = K_1 y_2, \quad u_2 = K_2 y_1, \quad t \geq 0$$

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Let $K_1 = k_1 I$, $K_2 = k_2 I$, $t_{l+1} = t_l + \Delta$.

Setting $\Delta = 0.2$ and by solving LMIs (25) and (26) in Theorem 3, we get $k_1 = k_2 = -0.01$, $c_1 = c_2 = 0.85$, $c = 1$, $d_1 = 4$, and $d_2 = 3$.

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$\Rightarrow$ by Theorem 3, under the designed interconnection control, the network is stabilized to ISS.

Thank you for your attention!